Black-Scholes-Formula

1. Parameters

$S_0 \ = \ underlying\ price\ in\ {}^{\$}/_{share}$

$X \ = \ strike\ price\ in\ {}^{\$}/_{share}$

$\sigma \ = \ anualized\ volatility\ {}^{\%}/_{p.a.}$

$r \ = \ current\ risk - free\ interest - rate\ {}^{\%}/_{p.a.}$

$q \ = \ dividend\ yield\ {}^{\%}/_{p.a}$

$t \ = \ time\ to\ expiration\ {}^{\%}/_{year}$

$T \ = \ number\ of\ days\ to\ expiration\ (not\ divided\ by\ days\ per\ year)$

2. Additional Functions

$e \ = \ exponential\ constant \ \rightarrow \ 2.7182818$

$N(x) \ = \ standard\ normal\ cumulative\ distribution\ function$

$ln(x) = natural\ logarithm$

3. Call & Put Options Price Formulas

$C = \ S_0 e^{-qt} * N(d1) - Xe^{-rt} * N(d2)$

$P = \ Xe^{-rt} * N(-d2) - S_0 e^{-qt} * N(-d1)$

$$d1 = \frac{\ln\left(\frac{S_0}{X}\right) + t(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{t}}$$

$d2 = d1 - \sigma\sqrt{t}$

4. Black-Scholes Formulas for Option Greeks

1. Delta:

$$\frac{\partial C}{\partial S} = \Delta_{Call} = e^{-qt} * N(d1)$$

$$\frac{\partial P}{\partial S} = \Delta_{Put} = e^{-qt} * (N(d1) - 1)$$

2. Gamma:

$$\frac{\partial'' C}{\partial'' S} = \gamma = \frac{e^{-qt}}{S_0 \sigma \sqrt{t}} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d1^2}{2}}$$

3. Theta:

$$\frac{\partial C}{\partial t} = \theta_{Call} = -\frac{S\phi(d1)\sigma}{2\sqrt{t}} - rKe^{-rt}N(d2)$$

$$\frac{\partial P}{\partial t} = \theta_{Put} = -\frac{S\phi(d1)\sigma}{2\sqrt{t}} + rKe^{-rt}N(-d2)$$

$$where\ \phi(d1) = \frac{e^{-\frac{d1^2}{2}}}{\sqrt{2\pi}}$$

4. Vega:

$$\frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = \nu = S_0 e^{-qt}\sqrt{t} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d1^2}{2}}$$

5. Rho:

$$\frac{\partial C}{\partial r} = \varrho_{Call} = Xte^{-rt} * N(d2)$$

$$\frac{\partial P}{\partial r} = \varrho_{Put} = -Xte^{-rt} * N(-d2)$$